Beta spirals and absolute velocities in different oceans

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Abstract—A simple steady-state theory of the geostrophic velocity field in the main thermocline of large-scale ocean gyres predicts the existence of current spirals associated with either the vertical component of velocity or local density change due to heating or cooling. The sense of rotation of the horizontal velocity vector with increasing depth is anticyclonic where the vertical component of velocity is downward and it is cyclonic where the vertical component is upward or where there is local cooling. These two cases correspond to what is expected from the Ekman pumping and climatic influences in subtropical and subpolar gyres, respectively.

A technique is devised for computing the absolute three-dimensional field of velocity from observed density data alone, assuming further immiscibility of the density stratification and simple linear beta-plane vorticity conservation.

Hydrographic station data in six mid-ocean areas, smoothed to remove meso-scale eddy noise, reveal spirals in the main thermocline that correspond to predictions. In the subtropics of the North Atlantic, South Atlantic, and North Pacific, a consistent picture of upper-level equatorward and westward flow with smaller, deeper flows in an opposite direction emerges. Vertical downward flows are found in the top of the main thermocline, presumably as deep extensions of the surface Ekman forcing. A case of a subpolar gyre (North Atlantic) appears to be a strong spiral dominated by cooling rather than the vertical velocity associated with the Ekman layer.

Indications in the treatment suggesting departures of the real ocean from the conditions of the model are discussed.

1. INTRODUCTION

1.1 Framework of the problem

The classical problem of physical oceanography has been to obtain the distribution of velocity from the observed density field by the assumption of geostrophy. As is well known this method gives only the vertical shear of the horizontal velocity, which upon vertical integration produces an integration constant that must be determined by some further constraint. The book by Fomin (1964) summarizes some of the early methods, which are mostly intuitive and cannot be justified satisfactorily. One method that can be framed in quantitative terms is the application of conservation of certain properties, such as potential temperature, to flow in and out of a closed volume. Hidaka (1940) attempted to apply conservation of salt and heat in small volumes bounded by quadrilaterally arranged sets of station data, but the examples he presented have been shown to be numerically ill-conditioned. Even Wunsch's (1977) reformulation of Hidaka's method in terms of the powerful 'inverse method of geophysics' leads to an infinity of possible solutions that must be narrowed down by additional constraints (Wunsch's method allows convenient addition of these constraints and techniques for evaluating their effectiveness). In the end, however, the

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The method presented here and in a preliminary discussion (STOMMEL and SCHOTT, 1977) also depends upon the conservation of some property, such as density, assuming that cross-isopycnal mixing is negligible. It also assumes that in the interior of the ocean the simple linear conservation statement that $f \partial p / \partial z$ remains constant along flow lines holds. This is the same as asserting that the potential vorticity is conserved and that the relative vorticity is small. Thus the method cannot be expected to hold in regions of strong current with large relative vorticity. In practice it seems to be applicable only to mid-ocean regions where there is an appreciable north-south component of the geostrophic velocity. It is a steady-state local calculation, determining the three components of velocity as functions of depth at a single geographical location. It uses the density data only and does not require information about wind-stress at the surface, about the slope of the bottom topography, or details of frictional boundary layers at the bottom. It does require substantial hydrographic station data surrounding the point in question to compute slopes of the density surfaces with the

Fig. 1. Dynamic topography of the 100-dbar level relative to 1500 dbar for the North Atlantic with positions of points where spirals were analysed.
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Fig. 2. Relative current spirals for points A, B, and C. At point B spirals for different sections used.
Depths in 100 m.

‘noise’ (due to ubiquitous eddies) filtered out. We have confined our calculations to locations where it may be hoped that large-scale long-period variability of the density structure is minimal. A large enough data base to determine the amount of such contamination by variability on scales other than the eddy scale does not yet exist in the centers of subtropical oceanic gyres. Our method involves writing down a number of linear algebraic equations for the absolute reference velocities $u_0$ and $v_0$. The system is modestly overdetermined, so that an estimate can be made of how well the various constraints are satisfied. It would be interesting to know how much of this violation of the various constraints is due to physical processes such as mixing across isopycnal surfaces, deviations from geostrophy, and influence of eddy-terms on the vorticity balance, or to inadequate determination of the mean slopes from the
limited data set. Definitive answers must be deferred until an adequate data base has been acquired.

Because it is not generally recognized how common spirals actually are in geostrophic velocity profiles through the main thermocline, it is useful to illustrate a few such spirals in relative velocity in the North Atlantic Ocean. Figure 1 shows their locations A, B, and C in the interior of the North Atlantic Ocean on a chart of the dynamic topography of 100 dbar relative to 1500 dbar in dynamic centimeters. Point A is in the North Equatorial Current, point B is in the center of the subtropical gyre at the latitude where the relative velocity appears to be largely meridional and southward, and point C is in the subpolar gyre where there is a substantial northward component of the relative velocity.

Figure 2 shows hodographs of the relative velocities at these three points. As explained in later sections referring individually to these three regions, the velocities are computed by smoothing over a moderate number of stations rather than from three or four individual stations. This smoothing is necessary because of the noise introduced by meso-scale eddies.

There are marked spirals in the relative velocities at all three points. Shallow levels are omitted at points A and B, and it is believed that the mixed layer never penetrates to the levels shown in the figures. In the case of point C, however, wintertime mixing can be very deep—perhaps as much as 600 m, so this spiral may not be truly geostrophic. The direction of turning of the velocity vector with depth for points A and B is to the right with depth as is normal for Ekman spirals too, but in the case of point C the sense of rotation with depth is reversed.

1.2 The causes of a spiral in the geostrophic velocity field

The equations of steady-state heat transport by advection, for vertical geostrophic shear and for the linear vorticity conservation are, in conventional notation:

\[ u \rho_x + v \rho_y + w \rho_z = \dot{\rho} \]  \hfill (1.1)

\[ u_z = \gamma \rho, \quad v_z = -\gamma \rho, \quad \gamma = g/\rho_0 \]  \hfill (1.2)

\[ \beta v = f w_z. \]  \hfill (1.3)

The term \( \dot{\rho} \) denotes density increase by a local in situ source of cooling, which we neglect at depth in the subtropics. Insert the equations (1.2) into (1.1),

\[ -u w_z + v u_z = \gamma ( -w \rho_z + \dot{\rho} ) \]

and write the velocities in polar form

\[ u = V \cos \theta, \quad v = V \sin \theta. \]

We then obtain the equation for the rate of turn of the vector direction with \( z \) (positive upwards):

\[ \theta_z = \frac{\gamma}{V^2} (w \rho_z - \dot{\rho}). \]  \hfill (1.4)

Here \( V^2 \) is positive, \( \rho_z \) is negative, and \( \gamma \) depends upon the hemisphere (positive in the northern). The direction of rotation therefore depends essentially upon the sign of \( w \) and \( \dot{\rho} \) because the signs of the other factors are fixed by geography: \( \gamma \) is positive in the northern hemisphere, negative in the southern, \( V^2 \) is positive definite, and \( \rho_z \) is universally negative for static stability. In the absence of vertical velocity \( w \) and \( \dot{\rho} \) there would be no turning of the
geostrophic velocity with depth. Subsurface heating ($\dot{\rho} < 0$) or downward vertical component of velocity ($w < 0$), such as might be expected in a subtropical gyre, lead to positive $\theta_z$ in the northern hemisphere, a direction of turning of the geostrophic velocity in the same sense as that of the Ekman spiral. On the other hand, regions of subsurface cooling or upward vertical component of velocity produce a direction of spiralling opposite to that of an Ekman spiral. In low latitudes seasonal variations on the depth of the surface mixed layer do not extend below 200 m, so one can assume $\dot{\rho} = 0$ and that any spiralling must be due to $w$. In subpolar regions wintertime vertical mixing (with $\dot{\rho} > 0$) can extend to depths as great as 600 m, so that the $\dot{\rho}$ effect may dominate over that of $w$.

If we now confine ourselves to considering the implications of equation (1.4) on spiralling in the main thermocline of the subtropics, we see how important the additional constraint given by equation (1.3) is [equation (1.3) is not used in deriving equation (1.4)]. If at any depth there is a meridional component of velocity $v$, then by equation (1.3) $w$ cannot vanish everywhere and there must be some spiralling. If, on the other hand, the flow field is entirely zonal, $w$ must be a constant with depth and might be zero. In this case it is possible to have a degenerate non-spiral zonal flow. Our understanding of the general oceanic circulation suggests that meridional components of flow must be forced by the Sverdrup relation. It is therefore in the main thermocline of the central regions of subtropical gyres, where strong Sverdrup forcing exists, that we make our first search for the spiralling of the geostrophic velocity. Order-of-magnitude considerations suggest appreciable spiralling. Thus substitution of characteristic values ($V = 1 \text{ cm s}^{-1}$, $w = 3 \times 10^{-5} \text{ cm s}^{-1}$, $\rho_z = 3 \times 10^{-8} \text{ cm km}^{-1}$, $\gamma = 10^7 \text{ cm s}^{-1}$ yields a rate of turn of 1 rad. Because beta vorticity dynamics play such an important role in determining the structure of the vertical velocity component, we call these spirals beta spirals.

We now proceed to a more orderly way of introducing the beta constraints into the problem and devise a method for calculation of the absolute velocity field.

1.3 Method for determination of the absolute velocities

In the following we confine ourselves to the stratified ocean and assume $\dot{\rho} = 0$. If $h(x, y)$ represents the height (measured upwards) of a density surface in the ocean, we can write the following equivalent of equation (1.1):

$$uh_x + vh_y = w.$$  
(1.1')

Taking the $z$ derivative and using equation (1.3), we obtain

$$(uh_x + vh_y)_z = \frac{\beta}{f}v.$$  
(1.5)

The equations (1.2) can be expressed in terms of $h$ also:

$$u_z = -\gamma h_y \rho_z, \quad v_z = \gamma h_x \rho_z.$$  
(1.2')

Substituting these into (1.5) we obtain:

$$uh_{xz} + v(h_x - \beta z/f)_z = 0.$$  
(1.6)

which holds for different density surfaces at many depths. Since $h_{xz}$ and $h_{yz}$ can be determined from the observed density field at any mid-ocean location, the direction of the absolute velocity, $v/u$, is known as a function of depth. If either coefficient (of $u$ or $v$) vanishes without the other vanishing at some particular depth one of the components of the absolute velocity
also vanishes there, thus fixing the 'depth of no motion' in that direction. Because of eddy noise these coefficients cannot (with the present data) be very well determined at any one depth, and therefore the absolute velocity can be determined better by simultaneous use of data at all depths instead of by location of the depth where the coefficients individually vanish. We will use the information contained in the main thermocline, excluding depths near the surface where unknown stresses and heating-cooling occur. Thus we introduce the notation $u = u_0 + u', v = v_0 + v'$, where the primes represent the relative geostrophic velocity with respect to a reference level where the absolute velocities are $u_0$ and $v_0$. Equation (1.6) can then be rewritten in the form:

$$u_0 h_{xz} + v_0 (h_{yz} - \beta/f) + u' h_{xz} + v' (h_{yz} - \beta/f) = r = 0.$$  

Writing this for several depths yields an overdetermined system for $u_0$, $v_0$. A 'best-fit' corresponds to the solution for which the sum of the squares of the residuals, $R$, is least. Sudo (1965) used this relation in a form vertically integrated to the Ekman layer as a top boundary condition, thus introducing the uncertain specification of the mean wind-stress; he also did not attempt to smooth out eddy noise.

Once the values of absolute $u$ and $v$ have been obtained, they can be used in equation (1.1), together with observed $h_x$ and $h_y$ values, to calculate $w$, which we should expect to match the independently known (from wind-stress) Ekman pumping near the surface and to approach a small value appropriate to the amplitude of the abyssal circulation at the bottom of the main thermocline.

## 2. Method of Analysis

### 2.1 The calculation of the absolute currents from the set of equations

We have to determine $u_0$, $v_0$ from our $N$ equations

$$a_j u_0 + b_j v_0 = c_j \quad j = 1 \ldots N$$

or in matrix notation

$$A u - c = 0,$$  

where $u = (u_0, v_0)$ and $A$ is the matrix of the coefficients $a_j$, $b_j$. The solution of (2.1) means to determine $u_0$, $v_0$ such that the sum of the squared residuals

$$R = (Au - c)(Au - c)^*$$

is a minimum.

The solution of the minimum condition is

$$u = (A^* A)^{-1} A^* c,$$  

where $A^*$ is the transpose of $A$ and $(A^* A)^{-1}$ is the inverse of $A^* A$.

If all equations are independent, the standard deviation of the result is,

$$\delta = \left\{ \frac{R}{N - 2} \right\}^{1/2}$$

and the standard deviations on the calculated parameters are

$$\delta_k = \delta \cdot \left\{ (A^* A)_{kk}^{-1} \right\}^{1/2} \quad k = 1, 2,$$  

where \((A^*A)^{-1}\) is a diagonal element of the inverse matrix. Under the assumption that the data are normally distributed, the 95% confidence bars on the parameters are

\[ u_k = t_{95,N-2} \cdot \delta_k, \]

with \(t\) the t-distribution, with \(N - 2\) degrees of freedom. This standard method always yields a result for our set of equations and the question is whether the result is a significant one. Posing this question the other way around, are the residuals significantly different from zero or not?

2.2 The interpretation of the results

(a) The quality of the residual. The magnitude of the equations is given by the sum of the squared inhomogeneous terms

\[ C = \sum_j c_j^2. \]

For any reasonable fit one will have to expect that the sum of the squared residuals,

\[ R = \sum_j r_j^2, \]

is much smaller than this number, i.e. \(R/C \ll 1\).

Different equations of the system may contribute very different fractions to the residual \(R\). A few equations may dominate the residual such that \(R/C \ll 1\) but a number of small magnitude equations may still be unsatisfied. Hence, besides \(R/C \ll 1\) one must check whether \(|r_j/c_j| \ll 1\) for all values of \(j\). But the question remains how small the residual has to be to indicate a significant fit. If we have a good spatial and temporal coverage of the density field we can calculate the noise on slopes and geostrophic currents and from there on get quantitative measures of the significance of the fit. The reality, however, is that the coverage of ocean areas with hydrographic stations is not so dense in time and space as one might think. Our data consist of a few sections only in any area and consequently our estimates of noise will be poor.

An obvious problem in the judgement of the residual is the reference level chosen. A change of the reference level with a change of the absolute currents by \(\Delta u_0, \Delta v_0\) will not change the residuals \(r_j\), but will change the \(c_j\) by

\[ \Delta c_j = -h_{xz,j} \Delta u_0 - (h_{yz,j} - \beta/f) \Delta v_0 \]

and hence the quality number of the fit, \(R/C\). Therefore this number is of only relative importance and must be compared with an estimate of the noise or with results for wrong values of \(\beta/f\).

(b) Noise considerations. Let us assume a model would fit the data exactly, except for scatter on the data. The variance due to that scatter is

\[ n_j^2 = \delta^2(u_j, h_{xz,j}) + \delta^2 \left[ v_j \left(h_{yz,j} - \frac{\beta}{f}\right)\right]. \tag{2.4} \]

If there is no systematic deviation of the measured data from the proposed model then the residuals \(r_j\) cannot be significantly larger than the scatter on the data, \(n_j\). The test is whether...
\( R = \sum r_j^2 \) is equal to \( N = \sum n_j^2 \) within confidence bars. If we assume the residuals to be normally distributed, which should be a fair assumption, the test on \( R \) and \( N \) uses the \( \chi^2 \) distribution. If the noise is well known, then the test is readily calculated. Assuming again the \( r_j \) to be normally distributed yields confidence bars on \( R \) given by the two-sided \( \chi^2 \) distribution for the number of degrees of freedom (NDF).

If the noise is well known, the case \( R \ll N \) cannot exist. If it does, it indicates an overestimation of noise. If \( R \gg N \) the model has to be rejected or the noise is underestimated, or both.

The proper way to proceed with the analysis would be the normalization of the equations with their noise, because it would enhance 'good' equations and reduce the contribution of supposedly bad data. This procedure would require that the approximate errors of \( u_0, v_0 \) be known at the beginning of the analysis, but this problem could be overcome by iteration. The test would be whether

\[
F = \frac{1}{N-2} \sum \frac{r_j^2}{n_j^2} = 1
\]

within confidence bars, which are again determined from \( \chi^2 \) with NDF.

We did some calculations with noise estimates, but we found that we did not know the noise well enough really to use the described test for the decision whether the model fitted a data set significantly or not. Therefore, we did not use noise normalization. Hence, in the following, we will only give some estimate of errors on slopes to provide a feeling of how well the data are being determined.

(c) \textit{Normalization by the inhomogeneous terms.} Normalization of all equations by \( c_j \) would provide all equations with about the same weight and prevent a few large equations from dominating the result. But now the residuals, \( r_j \), would depend on the arbitrary reference level, which they did not before. Furthermore, this method would especially enhance the deep equations with their small relative currents, \( u_j, v_j \) and hence would introduce an unwanted bias onto the fit.

(d) \textit{The method of wrong latitude as a practical way out.} Equation (1.6) would be homogeneous without the \( \beta / f \) term and would yield only the ratio of absolute currents. Hence, our result must depend crucially on the \( \beta / f \) term.

We can expect, for a good fit, that residuals \( R \) are larger compared with \( C \), if we insert wrong latitudes into the calculation of \( \beta / f \). Thus we calculate \( R/C \) for all latitudes between the South and North poles as a check whether the smallest value is at the correct latitude. At the equator we insert a small latitude, say 1', to avoid the singularity.

One could extend this method of wrong latitude to the \( f \)-term in the geostrophic currents, correspondingly. But as we see from (1.6) and (1.7), this would multiply the \( c_j \) term by a constant factor and yield \( u_0, v_0 \) with a corresponding correction but would not change the \( R/C(\phi) \) curve.

(e) \textit{Number of degrees of freedom and error bars.} The profiles of relative currents and slopes are smooth functions and not all of the equations can be expected to be independent. Hence, NDF is smaller then \( N - 2 \), sometimes much smaller.

To get an idea of the number of independent equations, we plot the \( a_j u_0 + b_j v_0 \) and \( c_j \) against depth and see how many points one needs to describe these profiles. Then we take this corrected NDF and calculate the standard deviations in (2.3). This method is not the correct one to determine error bars for fits to non-independent data, but it should at least provide an estimate of how the different results compare.
3. THE CENTRAL NORTH ATLANTIC GYRE AT 28° N, 36° W

As mentioned above, it can be expected that the strongest beta effect will be observed in regions of strongly forced meridional flow. For this reason, the point B at 28° N, 36° W in the North Atlantic was chosen. Its position within the field of dynamic topography of the subtropical gyre is central. This point was used in our preliminary paper to illustrate the principles of the method (STOMMEL and SCHOTT, 1977).

3.1 Data selection

To obtain the slopes of the isopycnals at this central point, it was necessary to use stations over a fairsized area (Fig. 3) so that eddy noise could be smoothed out. There is a problem in the choice of the size of this area: it should be small enough compared to the gyre so that it will represent a point in the gyre, but large enough to contain enough observations to establish the mean slopes in the presence of the eddy noise. The choice is severely constrained by the sparsity of data available. Because we were also concerned about the quality of the data used, we used stations made during the I.G.Y., appearing in FUGLISTER'S atlas (1960): Discovery Stas. 3594 to 3607 at 24° N, Atlantis Stas. 5754 to 5765 at 28° N, and Discovery Stas. 3627 to 3641 at 32° N. These three sections, with slopes estimated by eye are shown in Fig. 4. These are the slopes used by STOMMEL and SCHOTT (1977). The high amount of eddy noise is clear from these sections.

The slopes $h_x$ at the central point can be computed in two ways: from the 28° N section
Fig. 4. Density sections at 32°N (Discovery 3627 to 3641), at 24°N (Discovery 3594 to 3607) and 28°N (Atlantis 5754 to 5765). The heavy lines are estimated fits to isopycnals.

alone, or from an average of the 24 and 32°N sections. The \( h_y \) slopes can be computed only from the difference between data on sections at 32 and 24°N. An alternative way to fitting lines by eye is to read depths of isopycnals from the sections at intervals of 100 km and then to make a numerical fit by linear regression.

In Fig. 5(a) and 5(b) the values of the slopes are shown as functions of depth, computed in both ways. The term 'estimated' means the line is drawn by subjective judgement; the term 'from line fits' means that linear regressions are computed for each isopycnal using depths of the isopycnals read from the sections at intervals of 100 km. Standard error bars of the calculated slopes are indicated for some depths.

The distribution of slope \( h_y \) with depth (Fig. 5b) is much the same for both methods of calculation—within the error bars except at the 900- to 1000-m depths. On the other hand, there are substantial differences in the curves that give \( h_y \) as a function of depth, \( z \). The main difference depends upon whether the Atlantis 28°N section is used or not. As can be seen from Fig. 5(a) use of this section causes a narrower maximum at mid-depths and smaller values of \( h_y \) in the range 400- to 900-m. This difference is larger than errors introduced by the meso-scale eddies, and we cannot account for it. Considering, however, that the 28°N section was made in February 1959, whereas the 32 and 24°N sections were made in December and October 1957, these differences might be due to long term variations in the structure of the main thermocline. Only further data can establish the truth. Therefore we make our calculations both using 28°N and not.
In Stommel and Schott (1977) we used the slope data determined by graphical line fits to the isopycnals. Because of the inconsistencies discussed in that paper we redid the slope calculations by regression and present the results of both analyses here.

Fig. 5. Slopes in area B. (a) Slopes $h_x$ determined from the 28°N section (solid lines) and as averages from the 24°N and 32°N sections. Curves $h$ are from fitting lines visually to isopycnals, curves $h^*$ are from linear regressions. (b) Slopes $h_y$ determined from 24 and 32°N sections from graphical line fits (dashed) and from line regressions (solid). (c) Same as (b) but for $h_y - (\beta/f)z$. 
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Table 1. Slopes and relative currents for area B, 28° N, 36° W (slopes estimated graphically).

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<tr>
<th>depth (m)</th>
<th>( h_y ) (m/1000km)</th>
<th>( h_x ) from 28°N (m/1000km)</th>
<th>( h_x ) from 24°/32°N (m/1000km)</th>
<th>rel. currents for ( h_x, h_y ) from 28°N (m/sec)</th>
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Table 2. Slopes and relative currents for area B, 28° N, 36° W (slopes from linear regressions).

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3.2 The current spirals

The currents, relative to 1000 m, were calculated using equation (1.2'). The value of \( \rho_z \) was calculated in two ways: (1) from the 28°N section, or (2) as an average of the 24 and 32°N sections, consistent with the way \( h_x \) was calculated.

The slopes and relative currents are given in Table 1 for the estimated data and in Table 2 for those from linear regressions. The relative spirals are shown in Fig. 2 for the data from Table 1 and in Fig. 6 for those from Table 2. The relative spiral where 28°N is not used is larger in the north-south direction. The major difference between the two spiral sets in Figs. 2 and 6 is the effect of the deep positive \( h_y \) slope, which causes a reversal of the spiral rotation below 750 m.

3.3 The \( u_0 - v_0 \)-equations

The lines \( a_1u_0 + b_1v_0 = c_1 \), when plotted into the \( u_0, v_0 \)-plane, should all intersect in one point. In Fig. 7(a), the lines are given for every 100 m between 250 and 950 m for the data from
slope estimates using the 28°N section. Without further analysis, we see that there must be a tendency in our result depending on the depth range used in the analysis: the range 200 to 500 m will yield an intersection for \( u_0 \) between 0.3 and 0.4 cm s\(^{-1}\), for \( v_0 \) around 0.1 cm s\(^{-1}\), whereas the range 600 to 1000 m will yield a result close to \( u_0 = 0, v_0 = 0 \). In Fig. 7(b) the lines are shown for the data resulting from the regressions. A similar tendency is obvious. If we take the lines for the data from sections 24° and 32°N (Fig. 7c) the tendency for a grouping of intersections remains, but the deep intersections now are at negative \( u_0 \)-values.

3.4 Results

(a) *For the data including the 28°N section.* The results for the different data sets and depth ranges in each data set are given in Table 3.

At first we will consider the data set from Table 1, i.e. the slopes estimated visually and used in our previous presentation.

The best fit values for the case with the \( h_x \) and \( \rho_x \) determined from the 28°N section, when all levels from 200 to 1000 m are used, are \( u_0 = 0.34 \pm 0.04, v_0 = 0.066 \pm 0.022 \) cm s\(^{-1}\) (Fig. 5a). Also shown are lines of equal residuals, normalized by the minimum, which indicate, for example, that the residual will be about five times the minimum value if a zero or slightly negative \( v_0 \) is assumed.
Beta spirals and absolute velocities in different oceans

B. 28°N 36°W
with 28°N, estimates

B. 28°N 36°W
with 28°N, fits
Taking only the depth range 200 to 700 m yields \( u_0 = 0.37 \pm 0.02, \ r_0 = 0.11 \pm 0.02 \). The depth range 600 to 1000 m yields zero currents within the error bars: \( u_0 = 0.050 \pm 0.062, \ r_0 = 0.013 \pm 0.014 \text{ cm s}^{-1} \) (Fig. 7a). The above-mentioned tendency is confirmed by these results.

When the equations are normalized by \( c_p \), the result is \( u_0 = 0.039 \pm 0.032, \ r_0 = 0.007 \pm 0.007 \), i.e. the deep level result is imposed on the whole depth range because now the deep levels get increased weight in equation (2.1).

The results do not change too much when the data from line fits are used instead of the estimated slopes (Table 3b). For the total depth range \( u_0 = 0.22 \pm 0.08, \ r_0 = 0.042 \pm 0.027 \), and the deep level result again includes \( u_0 = 0, \ r_0 = 0 \) in the error bars, which is interesting because the data sets differ only in the deep levels due to the differences in deep \( h_p \).

An interesting feature is that the depth of no zonal motion of 550 m is the same (Table 3) for the 200- to 1000-m results of both data sets even though \( u_0 \) for the line fit data is only 65% of that for the estimated data. (If \( u_0 = 0.34 \) from the estimated data results were inserted instead of \( u_0 = 0.22 \), the level of no zonal motion would be shifted upward to 420 m for the line fit data.)

We can compare our results for the depths of vanishing zonal and meridional motion (Table 3) with what (1.6) prescribes for these depths from the vertical profiles of \( h_v = (\beta/f)z \) and \( h_{cv} \).
For the estimated slopes, Figs. 5(a) and 5(c) show that $h_x = 0$ and $[h_y - (\beta/f)z]_z = 0$ both at about 300-m depth, which makes (1.6) indeterminate there. Our result for no zonal motion at about 550 m is not in agreement with the slopes in Fig. 5(c). The situation looks better for the line fit data. Around 500 m the $[h_y - (\beta/f)z]$ curve has a minimum and $\alpha_z \neq 0$, in reasonable agreement with the best fit result for $u_0$.

The level of no meridional motion is 650 m for the estimated data when all depth levels are used, which does not correspond with the requirement from the $h_x$ profile in Fig. 5(a). For the line fit data we get 780 m, in agreement with the $h_x$ profile.

If the deep levels are left out, we get, for the 200- to 700-m range, 580 and 530 m, respectively. As this is about the same depth as that of vanishing zonal velocity, condition (1.6) is satisfied irrespective of the profiles $h_x$, $h_y - (\beta/f)z$.

The results for the deep levels alone, 600 to 1000 m, are also not contradictory to the slope profiles because $u$ and $v$ vanish at about the same depths.

To summarize: there is a general agreement between profiles and best fit results for the line fit data, but there are some disagreements for the estimated data.

(b) For the data without the 28°N section. The estimated slopes yielded $u_0 = 0.55 \pm 0.09$, $v_0 = 0.25 \pm 0.06$ cm s$^{-1}$, much larger values than the data including the 28°N section. The linear regression data from Table 2 give smaller deep velocities $u_0 = 0.26 \pm 0.23$, $v_0 = 0.14 \pm 0.08$ cm s$^{-1}$ for the depth range of 200 to 1000 m. The line fit results lead to a level of no zonal motion of 530 m and a level of no meridional motion of 730 m, much the same as for the data from the 28°N section.

The $h_x$ profiles show vanishing vertical derivatives at 300 to 400 m as do the $h_y - (\beta/f)z$ profiles (Figs. 5a, c). The result for no meridional motion is definitely not in agreement with the $h_x$ profile, which has $h_{xz} \neq 0$ here different from the $h_x$ profile from the 28°N section. The result for no zonal motion agrees with the profile of Fig. 5(c).

### Table 3. Results $u_0, v_0$ for area B with different depth ranges used.

<table>
<thead>
<tr>
<th>Depth range (m)</th>
<th>$u_0 \pm \delta u_0$ (cm/sec)</th>
<th>$v_0 \pm \delta v_0$ (cm/sec)</th>
<th>$R$</th>
<th>$C$</th>
<th>Depth (m) of no zonal meridional motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) with section 28°N, estimated slopes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200–1000</td>
<td>$0.34 \pm 0.04$</td>
<td>$0.066 \pm 0.022$</td>
<td>$0.038$</td>
<td>$3.7 \times 10^{-3}$</td>
<td>560</td>
</tr>
<tr>
<td>200–700</td>
<td>$0.37 \pm 0.02$</td>
<td>$0.11 \pm 0.02$</td>
<td>$3.7 \times 10^{-3}$</td>
<td>$530$</td>
<td>850</td>
</tr>
<tr>
<td>600–1000</td>
<td>$0.050 \pm 0.062$</td>
<td>$0.013 \pm 0.014$</td>
<td>$0.25$</td>
<td>$880$</td>
<td>900</td>
</tr>
<tr>
<td>200–1000</td>
<td>$0.039 \pm 0.032$</td>
<td>$0.007 \pm 0.007$</td>
<td>$0.56$</td>
<td>$900$</td>
<td>900</td>
</tr>
<tr>
<td>b) with section 28°N, slopes from line fits</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200–1000</td>
<td>$0.22 \pm 0.09$</td>
<td>$0.042 \pm 0.027$</td>
<td>$0.24$</td>
<td>$550$</td>
<td>780</td>
</tr>
<tr>
<td>200–700</td>
<td>$0.28 \pm 0.08$</td>
<td>$0.15 \pm 0.02$</td>
<td>$0.082$</td>
<td>$500$</td>
<td>530</td>
</tr>
<tr>
<td>600–1000</td>
<td>$-0.021 \pm 0.29$</td>
<td>$0.027 \pm 0.029$</td>
<td>$0.26$</td>
<td>$900$</td>
<td>850</td>
</tr>
<tr>
<td>c) without section 28°N, slopes from line fits</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200–1000</td>
<td>$0.26 \pm 0.23$</td>
<td>$0.14 \pm 0.08$</td>
<td>$0.51$</td>
<td>$530$</td>
<td>730</td>
</tr>
<tr>
<td>200–700</td>
<td>$0.40 \pm 0.13$</td>
<td>$0.41 \pm 0.11$</td>
<td>$0.062$</td>
<td>$530$</td>
<td>730</td>
</tr>
<tr>
<td>600–1000</td>
<td>$-0.39 \pm 0.20$</td>
<td>$-0.020 \pm 0.050$</td>
<td>$0.039$</td>
<td>$750$</td>
<td>950</td>
</tr>
</tbody>
</table>

*Out of depth range covered by data.
For the depth range 600 to 1000 m we get $u_0 = -0.39 \pm 0.20$, $v_0 = -0.020 \pm 0.054$, with levels of no motion below 1000 m, a result that needs more data for verification.

3.5 Residuals against depth, error bars

It is interesting to attempt to be explicit about how reliable the results for $u_0$, $v_0$ actually are. The values may be in error because the data are not representative of a steady state—as the two sets of $h_x$ based on whether the 28°N section is used suggests—or because the filtering out of observed meso-scale eddies is incomplete. The errors quoted above deal only with this second source of error.
To pursue the question of how well the 'best-fit' solution actually fits the data at all depths, the two sides of the \( u_0, v_0 \) equations are plotted as functions of depth. If the solution is a good fit, the two curves should correspond closely.

As an example of how the error bars in Fig. 7 and Table 3 were estimated we inspect once more the first case where:
Error bars calculated for the levels 200 to 1000 m were \( u_0 = 0.34 \pm 0.027, \quad v_0 = 0.06 \pm 0.016 \), assuming first that all equations are independent. The corresponding 95\% confidence bars are \( \Delta u_0 = 0.07, \Delta v_0 = 0.034 \). The equations versus depth, i.e. \( a_j u_0 + b_j v_0 \) and \( c_j \), are shown in Fig. 8(a). These profiles show that all of these equations are not independent. Equations for 250, 350, and 450 m are not predictable one from another, but this does not hold for the equations farther down. If we take the 950-m level and one intermediate level, say 650 m, then the 550-, and 750-, and the 850-m equations can be predicted within certain limits. Hence we can state that five equations are enough to describe the profile roughly. On the other hand, the 550-, 750-, and 850-m levels do contribute something because they are not totally predictable. It should be fair to assume that we got three degrees of freedom from eight equations.

An estimate of the change in error bars is possible by just correcting them for the lower NDF, which means a factor of \( \sqrt{2} \), i.e.

\[
\begin{align*}
  u_0 &= 0.34 \pm 0.04, \\
  v_0 &= 0.06 \pm 0.022.
\end{align*}
\]

The effect on the 95\% confidence bars is larger; they change to \( \Delta u_0 = 0.13, \Delta v_0 = 0.071 \), and the meridional velocity is no longer significantly different from zero.

When we omit levels from the calculation the NDF reduces further, to only one or two with a corresponding increase in error bars.

The \( a_j u_0 + b_j v_0 \) in Fig. 8(a) fit the \( c_j \) quite well in the upper part of the profile, but both differ considerably below 550 m. The ratio of the sum of the squared residuals, \( R \), to the sum of the squared \( c_j \), \( C \), is \( R/C = 0.038 \), but \( r_j \) is not \( \ll c_j \) for the deep levels.

The depth range 200 to 700 m gives a better fit with \( R/C = 3.7 \times 10^{-3} \), whereas the 600- to 1000-m range has a larger residual of \( R/C = 0.25 \). Also indicated in Fig. 8(a) is \( a_j u_0 + b_j v_0 \) for the case of normalization by \( c_j \), where the curve now should be close to 1 (see upper scale) for a good fit. There is no good correspondence and we do not pursue this method further.
Fig. 9. Residuals, normalized by C, versus latitude for depth ranges as indicated (reference level = 1000 m). (a) For estimated data, using the 28°N section. (b) For linear regression data, not using the 28°N section. (c) Same as (b) but for reference level 200 m.

The equations for the line fit data are more irregular (Fig. 8b) and would therefore allow a somewhat higher estimate of NDF, but we took the same numbers for the error bar correction in Table 3 as for the estimated data. The $R/C$ values [Table 3(b)] are larger than the previous ones.
For the data with the 28°N section (Figs. 8a, b) there seems to be at least a reasonable correspondence between both sides of the equations over the total depth range. However, for those line fit data using the average of the 24/32°N sections, instead of the 28°N section, the \(a_j u_0 + b_j v_0\) differ considerably from the \(c_j\) when the total depth range 200 to 1000 m is considered. The residual is \(R/C = 0.51\) for this case. This cannot be a reasonable fit of the data by the model. It gets better when only the depth range 200 to 700 m is used; then \(R/C = 0.062\).

The problem about the plots of both sides of the equations against depth is that the residuals \(r_j\) do not depend upon the reference level, as stated before, but the \(a_j u_0 + b_j v_0\) and \(c_j\) do, which makes the profiles in Fig. 8 change with the arbitrary reference level. As an example, the case of Fig. 8(c) is repeated in Fig. 8(d) with the reference depth 200 instead of 1000 m. Now the \(c_j\) values are much larger because of the spiral shape (Fig. 2), and the differences between \(a_j u_0 + b_j v_0\) and \(c_j\) are small compared with \(c_j\) at almost all depths although they are the same as in Fig. 8(c). For this case, we get \(R/C = 0.019\) only.

Because of the stronger shears near the surface, deep reference levels give smaller values of \(C\) and are to be preferred to near-surface ones.

### 3.6 The method of wrong latitude

In a typical beta spiral the configuration of the density surfaces is tightly controlled by dynamical and kinematical constraints. For a given set of data it might be expected that only the correct value of beta (or latitude) will yield a very low value of the least square residual; a wrong value of beta presumably will yield higher residuals in (1.7) even for the best fit, than will the correct value. To test this idea the minimum squared residual normalized to the square of the right-hand side of the equations is computed for all latitudes from the South pole to North pole, and shown in Fig. 9. The 200- to 1000-m range gives a sharp minimum at the correct latitude. A similar curve is obtained for the 200- to 700-m range, but for 600 to 1000 m there are two minima and neither is at the correct latitude.

At least for the upper part we can state that the vorticity conservation model seems applicable. The density field 'senses' its latitude or, expressed in another way, the correct value of beta allows the fluid to flow almost freely without violating constraints.

For the line fit data, using the 28°N section, the \(R/C\) versus latitude curves have lower quality.

The line fit data using only the 24 and 32°N sections yield a minimum on the southern hemisphere for the 200- to 1000-m range, the top levels 200 to 500 m show a minimum close to the correct latitude, and the 400- to 1000-m range gives at least a secondary minimum at the correct latitude.

If we change the reference depth to 200 m the normalized residuals, \(R/C\), get smaller, as discussed before, but the minimum on the southern hemisphere remains.

### 3.7 The vertical component of velocity

Once \(u\) and \(v\) are known, \(w\) can be computed from the first equation and is shown in Fig. 10(a) for the cases of estimated data including the 28°N section (solid), for line fit data including that section (dashed), and for line fit data not using the 28°N section (dotted). All three profiles are computed for the \(u_0, v_0\) from depth range 200 to 1000 m.

Standard error bars are also given using a slope error of \(\delta h_x = \delta h_y = 10^{-5}\) in accordance with our earlier statements. The large error bars on the curve for data using the 24/32°N sections are due to the large errors on the \(u_0, v_0\) values (Table 3c) for these data.
The vertical velocity approaches a negative value near the surface as is expected from the wind stress distribution in this location, which requires Ekman pumping downward. At 200 m this downward pumping is computed to be between $-2$ and $-3 \times 10^{-5}$ cm s$^{-1}$.

The line fit data with 28° N (dashed) show positive $w$ somewhat larger than the error bars below 500 m, because of the larger $h_y$ (Fig. 5b), whereas the estimated data (solid) approach small values there.

When the $u_0, v_0$ in the calculation of $w$ for the three cases of Fig. 9 are not those determined from the depth range 200 to 1000 m, but the smaller values resulting for the depth range 600 to 1000 m (Table 3), then we get larger negative values of $w$ at 200 and 300 m (see Table 4 and Fig. 10b).

3.8 Discussion of results

(a) Absolute spirals. The absolute current profiles for the three cases analysed are shown in Fig. 11. There actually is no single depth of no motion. For the dotted curve (data including the 28° N section, slopes estimated) the $u$ velocity vanishes at about 560 m and the $v$ velocity at about 650 m. The minimum velocity is at about 580 m. The errors in the location of the origin,
Table 4. Values of $w (10^{-5} \text{cm s}^{-1})$ at 200 and 300 m.

<table>
<thead>
<tr>
<th></th>
<th>$w$ for $u_0, v_0$ from depth range 200-1000m</th>
<th>$w$ for $u_0, v_0$ from depth range 600-1000m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200m</td>
<td>300m</td>
</tr>
<tr>
<td>data with 28°N estimated slopes</td>
<td>-1.8</td>
<td>-1.8</td>
</tr>
<tr>
<td>data with 28°N regr. slopes</td>
<td>-2.5</td>
<td>-1.2</td>
</tr>
<tr>
<td>data without 28°N, regr. slopes</td>
<td>-3.1</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

Fig. 11. Absolute current spirals for the three cases of Fig. 9.

when expanded to approximately $\pm 0.1 \text{cm s}^{-1}$, as explained in the text (to convert to 95\% confidence limits and to allow for the statistical dependence of some of the $u_0, v_0$ equations) admit the possibility of having a simultaneous vanishing of $u$ and $v$ near 575 m.

The direction of turning of the horizontal velocity with depth is to the right (clockwise) at all depths in the relative spiral as above, but as drawn in Fig. 11, the absolute vector rotates clockwise only from 200 to about 450 m, where the rotation vanishes; below that depth the spiral rotates counterclockwise. Although we have no objective grounds to argue against this computed reversal of rotation at mid-depth, it intuitively seems strange, especially because the relative spiral is in all respects such a smooth function with a smooth rotation of single sign. With the line fit data, there is a change of rotation also in
the relative spirals, but the counter-rotating part of the spirals is small. The depths of minimum velocity are 580 m for the 28°N spiral and 640 m for the 24/32°N spiral. Defant's intuitive choice of 1100 m as the depth at which both $u$ and $v$ vanish would make the absolute and relative spirals coincide much more closely.

Our calculations for depth range 200 to 1000 m also give zonal velocities at 1200 m of about $+0.3 \text{ cm s}^{-1}$. It may seem odd to have a mean eastward flow at this depth where the high salinity Mediterranean Water seems to be flowing westward. Again we can offer no logical objection but only point out something curious.

Leetmaa, Niiler and Stommel (1977) computed meridional transport per unit depth across long sections at 24 and 32°N. These sections are those used in the present paper, but the data were used over a greater range of longitude. Leetmaa et al. (1977) found $v = 0$ at 800 m at 24°N and at 1000 m at 32°N. Because they did not use the 28°N data, their results should be compared with the 24/32°N spiral shown in Fig. 11 in which $v = 0$ at 730 m.

If we took the results from the depth range 600 to 1000 m then we would avoid these inconsistencies much easier. The 24/32°N results even yield the expected westward flow at 1000 m for this depth range. Also the depth tendency in Figs. 7(a),(b) in the results for the data using 28°N section seems to indicate that using deeper data than 1000 m might again yield westward velocity at 1000 m.

6. VERTICAL VELOCITY AND EKMAN PUMPING

The vertical component of velocity $w$, as computed by our method from the density field alone, should be expected to approach the value of the divergence of the Ekman flux—as computed from the distribution of wind stress over the area. Recent recomputation (Leetmaa and Bunker, 1978) of annual mean Ekman pumping, $w_e$, from wind observations over 2° latitude × 5° longitude areas near point B yield a range from $-8.2$ to $-18.8 \times 10^{-5} \text{ cm s}^{-1}$. The results for $w_{200}$ at 200 m (Table 4) range from $-1.8$ to $-6.7 \times 10^{-5} \text{ cm s}^{-1}$. Theoretically, these two vertical components are related by

$$w_e - w_{200} = \frac{\beta}{f} \int_{-200}^{0} v \, dz,$$

where $v$ is the absolute meridional component of the geostrophic velocity. To estimate this balance, the absolute profile (Fig. 37) was extrapolated to the surface by use of the section in Fig. 4, yielding an average $v$ between 0 and 200 m of $-1.4 \text{ cm s}^{-1}$, or a right-hand side $-8.3 \times 10^{-5} \text{ cm s}^{-1}$. This number may not represent the annual mean, because the upper layers vary with time of year. Within the range of variability and uncertainty, the balance obtains.

4. THE WESTERN ATLANTIC AT 20°N, 54°W

4.1 Data selection and slope calculation

A set of data which, at first, seemed to be quite convenient for our purpose was available from two north–south sections (Chain, Atlantis) and two east–west sections (Discovery, Crawford) in the southwestern North Atlantic (Fig. 3). The meridional section distance was 888 km, the zonal distance 730 km, the center point, A, at 20°N, 54°W. We started with the analyses as we had done for area B, i.e. we estimated $\sigma$, slopes visually from the four sections, took averages and so forth.

The $\sigma$, slope analysis however, did not yield good results in terms of the residual versus latitude test. This was because of the high eddy noise, which spoiled especially the very weak
Fig. 12. Sections of specific volume anomaly around point A (Fig. 3). (a) One of the west–east sections: *Discovery* at 24°N. (b) One of the north–south sections: *Atlantis* at 50°30' E.
zonal slopes. We then proceeded by working with group averages of anomalies of specific volume, \( \delta_{ST} \), (Fig. 12). The noise contamination is especially obvious in the zonal sections of Discovery and Crawford. We calculated mean \( \delta \)-profiles by averaging over each of the sides of the rectangle (Fig. 3) formed by the four sections. The slopes \( h_x, h_y \) (Table 5 and Fig. 13) were calculated from the zonal and meridional differences of these averages and from the vertical derivative, \( \delta_z \), which was calculated from the mean of the four group averages. Dynamic depth anomalies were treated similarly and geostrophic currents relative to the 1000-m level were calculated from these (Table 5). The relative spiral was presented in Fig. 2. The slope errors in Table 5 were calculated in the following way: lines were fitted to the \( \delta \)-values along each side of the rectangle in Fig. 3 for all depth levels from 100 to 1000 m. The errors on a slope then resulted from the errors on the mean values of \( \delta \) on opposing sides of the rectangle, where again, for simplicity, we assume the vertical derivative, \( \delta_z \), to be error free. The error bars on the meridional slopes are about the same as in area B, but the zonal slope errors are somewhat larger. This may partly be because smaller portions of the available sections, only 730 km, were taken for the group averages and line fits whereas in area B the slopes were determined from sections 2400 km long.

The large slope errors at the top level and inspection of Fig. 12 show that surface effects are of influence. But because the spiral in Fig. 2 is most pronounced between 100 and 300 m we do not skip the 100-m level.

Table 5. Slopes of isosteres, isotherms, and isohalines and relative currents in area A, 20°N 54°W.

<table>
<thead>
<tr>
<th>depth (m)</th>
<th>isosteres (h_x (m/1000\ km))</th>
<th>isotherms (h_x (m/1000\ km))</th>
<th>isohalines (h_x (m/1000\ km))</th>
<th>rel. currents (u' v' (cm/sec))</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>(213^{+7}_{-112})</td>
<td>(241^{+67}_{-112})</td>
<td>(14)</td>
<td>(-4.15) (-1.22)</td>
</tr>
<tr>
<td>200</td>
<td>(43^{+21}_{-112})</td>
<td>(-7^{2}_{-7})</td>
<td>(15)</td>
<td>(-4.25) (-1.56)</td>
</tr>
<tr>
<td>300</td>
<td>(59^{+40}_{-17})</td>
<td>(-127^{+14}_{14})</td>
<td>(30)</td>
<td>(-4.52) (-2.38)</td>
</tr>
<tr>
<td>400</td>
<td>(89^{+21}_{-243^{+14}})</td>
<td>(-243^{+14}_{14})</td>
<td>(35)</td>
<td>(-3.18) (-2.22)</td>
</tr>
<tr>
<td>500</td>
<td>(97^{+20}_{-255^{+11}})</td>
<td>(-255^{+11}_{11})</td>
<td>(50)</td>
<td>(-2.22) (-2.22)</td>
</tr>
<tr>
<td>600</td>
<td>(60^{+23}_{-226^{+10}})</td>
<td>(-226^{+10}_{10})</td>
<td>(60)</td>
<td>(-1.43) (-0.07)</td>
</tr>
<tr>
<td>700</td>
<td>(42^{+17}_{-195^{+8}})</td>
<td>(-195^{+8}_{8})</td>
<td>(70)</td>
<td>(-0.67) (-0.02)</td>
</tr>
<tr>
<td>800</td>
<td>(14^{+15}_{-136^{+14}})</td>
<td>(-136^{+14}_{14})</td>
<td>(80)</td>
<td>(-0.29) (-0.05)</td>
</tr>
<tr>
<td>900</td>
<td>(3^{+15}_{-76^{+14}})</td>
<td>(-76^{+14}_{14})</td>
<td>(90)</td>
<td>(-0.08) (-0.02)</td>
</tr>
<tr>
<td>1000</td>
<td>(-15^{+25}_{-20^{+16}})</td>
<td>(-20^{+16}_{16})</td>
<td>(100)</td>
<td>(0) (0)</td>
</tr>
</tbody>
</table>
Fig. 14. The equations in the $u_0, v_0$-plane. Best-fit results with standard deviations for different depth ranges (in 100 m) as indicated.

Fig. 15. Both sides of the equations versus depth with $u_0, v_0$ from best fits to data.
4.2 Results

The lines \( a_j u_0 + b_j v_0 = c_j \) are shown in Fig. 14 for every 100 m from 150 to 950 m. The best fit result for the full depth range is \( u_0 = 4.38 \pm 0.14 \text{ cm s}^{-1}, \ v_0 = 0.79 \pm 0.10 \text{ cm s}^{-1} \). To correct the calculated error bars, the vertical profiles of both sides of the equation (Fig. 15) have to be inspected. The \( c_j \) values are large and positive at 150 m, negative at 250 m, positive again at 550 m and stay small below. The profile could be described with the points 150, 250, 550, and 950 m and intermediate values could be interpolated. Hence, the estimated number of degrees of freedom is NDF = 2. The corrected result therefore is \( u_0 = 4.38 \pm 0.26, \ v_0 = 0.79 \pm 0.19 \text{ cm s}^{-1} \). The 95\% confidence bars now would be \( \Delta u_0 = 1.12, \Delta v_0 = 0.78 \text{ cm s}^{-1} \), i.e. \( v_0 \) barely significantly different from zero.

As in area B, we find again in Fig. 14 the tendency for the intersections of lines to yield smaller values of \( u_0, v_0 \), as we use deeper levels. The depth range 400 to 800 m yields \( u_0 = 2.20 \pm 0.27, \ v_0 = 0.39 \pm 0.08 \text{ cm s}^{-1} \); for the range 600 to 1000 m we get \( u_0 = 0.92 \pm 0.37, \ v_0 = 0.07 \pm 0.08 \text{ cm s}^{-1} \). The top levels, 100 to 500 m, yield \( u_0 = 4.53 \pm 0.17, \ v_0 = 0.66 \pm 0.16 \text{ cm s}^{-1} \). These levels contribute most to the fit and determine the result, as is also obvious from Fig. 15.

4.3 Discussion of results

The depths of no zonal and no meridional motion, derived from these different results, are given in Table 6. The slope curves in Fig. 13 show vanishing derivatives for \( h_z \) and \( h_v \) at the same depth, 400 m. At 200 m, \( h_z \) has a minimum that would correspond with the depth of no meridional motion found for the total depth range but the zonal current also vanishes at that depth. Thus the slope profiles are of no help for discriminating among results.

The residuals versus latitude curves (Fig. 16) are good for the top levels—shown here for 100 to 400 m—although the minimum is displaced a little equatorwards. The result for the

<table>
<thead>
<tr>
<th>parameter</th>
<th>depth range (m)</th>
<th>( u_0 \pm 6u_0 ) cm/sec</th>
<th>( v_0 \pm 6v_0 ) cm/sec</th>
<th>depth of no zonal meridional motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>anomaly of</td>
<td>100-1000</td>
<td>4.38 ± .26</td>
<td>.79 ± .19</td>
<td>200</td>
</tr>
<tr>
<td>specific</td>
<td>100-500</td>
<td>4.53 ± .17</td>
<td>.66 ± .16</td>
<td>-</td>
</tr>
<tr>
<td>volume</td>
<td>400-800</td>
<td>2.20 ± .27</td>
<td>.39 ± .08</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>600-1000</td>
<td>.92 ± .37</td>
<td>.07 ± .08</td>
<td>670</td>
</tr>
<tr>
<td>temperature</td>
<td>100-1000</td>
<td>1.50 ± .62</td>
<td>-.22 ± .25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100-500</td>
<td>1.02 ± .17</td>
<td>-.105 ± .67</td>
<td></td>
</tr>
<tr>
<td></td>
<td>400-800</td>
<td>1.20 ± .78</td>
<td>-.19 ± .55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>600-1000</td>
<td>.47 ± .07</td>
<td>-.08 ± .03</td>
<td></td>
</tr>
<tr>
<td>salinity</td>
<td>100-800</td>
<td>3.05 ± .51</td>
<td>.15 ± .05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100-500</td>
<td>3.93 ± .32</td>
<td>.24 ± .37</td>
<td></td>
</tr>
<tr>
<td></td>
<td>400-800</td>
<td>2.15 ± .72</td>
<td>-.36 ±1.30</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 16. Residuals, normalized by C, versus latitude from depth ranges as indicated (reference level = 1000 m).

total depth range, 100 to 1000 m, still yields the minimum at the correct latitude. When the 100-m level is left out, the minimum is displaced equatorwards by 10°. The deep levels alone, 400 to 800 m and 600 to 1000 m, yield curves with minima at wrong latitudes.

We can thus conclude that the strong spiralling close to the surface, despite mixed layer influences, is definitely determined by the $\beta/f$-term. Hence, one might be inclined to trust these results more. Wunsch, in an application of his ‘minimum barotropic energy’ method to the Atlantis section at 50°30'W, also got eastward currents at 1000 m near 20°N (private communication). On the other hand, an eastward current of some 4 cm s$^{-1}$ at 1000 m is contrary to conventional expectations.

As in area B, the sections in area A stem from different seasons and years (Fig. 3). This makes it even more remarkable that the near-surface data yield good results of $R/C(\phi)$ in Fig. 16. Because of the inconsistencies in the results for different depth ranges we do not present upwelling velocities here.

4.4 Use of other hydrographic data

In the previous sections we used only density data for the analysis. The data contributed only a few degrees of freedom. If one had more measurements of different parameters, it should be possible to increase the number of independent equations. Taking temperature and salinity instead of density would not increase the NDF by the number of additional equations because both are correlated, but it would yield some new information.

To proceed with non-density data needs re-inspection of the equations. In equation (1.5) the term $u_x h_x + v_y h_y$ does not cancel out as it does for density by use of (1.2'). Hence, equation (1.6), for non-density conservative parameters, reads

$$u h_{xx} + v(h_y - \beta z/f)_x = -u_x h_x - v_y h_y$$

(1.6')
Fig. 17. Both sides of the equations, for temperature, versus depth, also $\Delta c_j = -u_j h_j - r_j h_j$.

Fig. 18. $R/C(\phi)$ for temperatures for depth ranges as indicated and for salinity (depth range 100 to 800 m).
and the simple arguments about zero current levels for vanishing slope derivatives are no longer applicable.

We used temperature, salinity, and oxygen for the analysis. The slopes of $T$ and $S$ are listed in Table 5, also calculated from group averages. The lowest two levels are left out for salinity, because $S_z \approx 0$ around 900 m. For the temperatures, the results are much smaller than for density, namely $u_0 = 1.50 \pm 0.62$, $v_0 = -0.22 \pm 0.33$ cm s$^{-1}$ for depth range 100 to 1000 m where we estimated NDF = 4 by inspection of the vertical profile of the components of the equation (Fig. 17). The tendency for the results to get smaller for deeper levels is also there but not so strong as with density data (Table 6).

The correction term $\Delta c_j = -u_z h_x - v_z h_y$ in (1.6') is also shown in Fig. 17. It makes a considerable contribution at some of the levels. The residual versus latitude curves for temperature data are shown in Fig. 18 for ranges 100 to 1000, 100 to 500, and 600 to 1000 m. The minimum for the total depth range is exactly at the correct latitude, but for the subranges two minima occur.

The salinity data yield different results (Table 6). They give a shallow minimum of the $R/C(\phi)$ curve close to the correct latitude (Fig. 18).

The differences in the results in Table 6 show that it is not possible to use the temperature and salinity slopes in a joint analysis to increase the NDF. The systematic differences may arise from mixing across temperature and salinity surfaces.
Oxygen data did not yield reasonable $R/C(\phi)$ curves and are not discussed here. Oxygen, as a non-conservative property, would also need a decay term in the equations.

5. THE SUBTROPICAL GYRE OF THE SOUTH ATLANTIC

The subtropical gyre in the South Atlantic is of interest for comparison with area B in the North Atlantic, especially with reference to the small vertical velocity found there.

Two sets of data can be used. There are the east–west sections occupied by Meteor in 1925 to 1927 (Fig. 19) (Wüst, 1957), and those occupied during the I.G.Y. (Fig. 19) (Fuglister, 1960).

We take the latter data first because the station distances are smaller and allow group averaging.

5.1 The I.G.Y. data

We worked with group averages of $\delta_{ST}$, the station groupings are indicated in Table 7. The longitudes of the group averages at each latitude were at about 15, 10, and 5°W. The mean $\delta$-profiles at points D(28°S, 10°W) and E(20°S, 10°W) are taken from the groupaveraging.

Table 7. Stations used for group averages of western, central, and eastern segments (see Fig. 19) of I.G.Y. data.

<table>
<thead>
<tr>
<th>Latitude (°S)</th>
<th>Station</th>
<th>West (av. longitude: 15°W)</th>
<th>Central (av. longitude: 10°W)</th>
<th>East (av. longitude: 5°W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16°S</td>
<td>Crawford</td>
<td>135 - 137</td>
<td>138 - 140</td>
<td>141 - 143</td>
</tr>
<tr>
<td>24°S</td>
<td>Crawford</td>
<td>435 - 437</td>
<td>438 - 441</td>
<td>442 - 444</td>
</tr>
<tr>
<td>32°S</td>
<td>Atlantis</td>
<td>5823-5825</td>
<td>5826-5827</td>
<td>5828-5830</td>
</tr>
</tbody>
</table>

Table 8. Slopes of isosteric planes and geostrophic currents at points D, E (from I.G.Y. data).

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>D. 28°S, 10°W</th>
<th>E. 20°S, 10°W</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_x$ (m/1000 km)</td>
<td>$h_y$ (m/1000 km)</td>
</tr>
<tr>
<td>100</td>
<td>7</td>
<td>-87</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
<td>-60</td>
</tr>
<tr>
<td>300</td>
<td>36</td>
<td>4</td>
</tr>
<tr>
<td>400</td>
<td>29</td>
<td>32</td>
</tr>
<tr>
<td>500</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>600</td>
<td>49</td>
<td>79</td>
</tr>
<tr>
<td>700</td>
<td>37</td>
<td>138</td>
</tr>
<tr>
<td>800</td>
<td>31</td>
<td>165</td>
</tr>
<tr>
<td>1000</td>
<td>28</td>
<td>184</td>
</tr>
</tbody>
</table>
Fig. 20. Spirals (relative to 1000 m) for points D and E.

Fig. 21. Slopes of planes of equal anomaly of specific volume at point D.

Fig. 22. The equations in the $u_0,v_0$-plane for point D. Best-fit results with standard deviations marked.
averages at 10°W north and south of these points, the meridional gradients from the differences of these two. The zonal gradients are from the differences between the group averages at 5 and 15°W at the latitudes of D and E.

The slopes and geostrophic currents calculated by this method are listed in Table 8 for points D, E. We did not calculate slope errors for these data.

The relative spirals (Fig. 20) turn in a sense opposite to that in the subtropical gyre of the northern hemisphere, which is to be expected for the absolute spirals; both are the same only if the deep velocities are found to be small.

5.2 Results for points D, E

The spiral effect is most pronounced at point D, which may raise expectations for good results, but the slope profiles, $h_x$ and $h_y - (\beta/f)z$, have several depths of vanishing vertical derivatives (Fig. 21).

The lines $a_i u_0 + b_i v_0 = c_j$ are shown in Fig. 22. There is no obvious tendency for deep lines to intersect more closely to the zero point than the shallow ones as at points B (Fig. 7) and A (Fig. 14).

The result for the depth range 200 to 1000 m is $u_0 = 0.38 \pm 0.08, v_0 = -0.18 \pm 0.04 \text{ cm s}^{-1}$ with an estimated NDF = 4 from inspection of both sides of the equations versus depth in Fig. 23. The curve of the normalized residuals $R/C$ versus latitude (Fig. 24) shows a deep minimum which is, however, displaced about 5° latitude towards the equator. Omitting the upper three or the lower three levels does not change the results drastically (Table 10).

This is the first data set so far where the deep levels alone yield reasonable curves of residuals versus latitude (Fig. 24). In fact, the absolute currents at 1000 m are quite similar to those in the northern gyre: towards east and poleward. The vertical velocity (Fig. 29), in the upper depth range, is also similar to that of area B (Fig. 10a): at 200 m, $w = -2 \times 10^{-5} \text{ cm s}^{-1}$, then follows a range of small $w$ between 400 and 700 m, then negative values below with $w =$
### Table 9. Slopes of isosteric planes and geostrophic currents at points F, G (from Meteor data).

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>F, 30°S, 15°W</th>
<th>G, 18.5°S, 15°W</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_x$ (m/1000 km)</td>
<td>$h_y$ (m/1000 km)</td>
</tr>
<tr>
<td>200</td>
<td>22</td>
<td>-10</td>
</tr>
<tr>
<td>300</td>
<td>27</td>
<td>40</td>
</tr>
<tr>
<td>400</td>
<td>30</td>
<td>61</td>
</tr>
<tr>
<td>500</td>
<td>32</td>
<td>68</td>
</tr>
<tr>
<td>600</td>
<td>37</td>
<td>73</td>
</tr>
<tr>
<td>700</td>
<td>39</td>
<td>76</td>
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<tr>
<td>800</td>
<td>43</td>
<td>61</td>
</tr>
<tr>
<td>900</td>
<td>45</td>
<td>46</td>
</tr>
<tr>
<td>1000</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>1100</td>
<td>41</td>
<td>125</td>
</tr>
<tr>
<td>1200</td>
<td>40</td>
<td>132</td>
</tr>
<tr>
<td>1300</td>
<td>37</td>
<td>139</td>
</tr>
<tr>
<td>1400</td>
<td>34</td>
<td>157</td>
</tr>
<tr>
<td>1500</td>
<td>34</td>
<td>157</td>
</tr>
</tbody>
</table>

### Table 10. Results for absolute currents at 1000-m depth at points D, E, F, G in the South Atlantic.

<table>
<thead>
<tr>
<th>Point</th>
<th>Depth Range (m)</th>
<th>$u_0 \pm \delta u_0$ (cm/sec)</th>
<th>$v_0 \pm \delta v_0$ (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (28°S, 10°W)</td>
<td>200-1000</td>
<td>.38 ± .08</td>
<td>-.18 ± .04</td>
</tr>
<tr>
<td></td>
<td>200-600</td>
<td>.43 ± .07</td>
<td>-.25 ± .03</td>
</tr>
<tr>
<td></td>
<td>400-800</td>
<td>.43 ± .13</td>
<td>-.14 ± .06</td>
</tr>
<tr>
<td></td>
<td>600-1000</td>
<td>.39 ± .03</td>
<td>-.08 ± .02</td>
</tr>
<tr>
<td>F (30°S, 15°W)</td>
<td>200-1400</td>
<td>.23 ± .43</td>
<td>-.11 ± .06</td>
</tr>
<tr>
<td></td>
<td>400-1000</td>
<td>.60 ± 1.25</td>
<td>-.06 ± .14</td>
</tr>
<tr>
<td></td>
<td>600-1400</td>
<td>.34 ± .36</td>
<td>-.05 ± .03</td>
</tr>
<tr>
<td></td>
<td>900-1400</td>
<td>.16 ± .10</td>
<td>.01 ± .02</td>
</tr>
<tr>
<td>E (20°S, 10°W)</td>
<td>200-1000</td>
<td>.43 ± .41</td>
<td>-.09 ± .06</td>
</tr>
<tr>
<td>G (18.5°S, 15°W)</td>
<td>200-1500</td>
<td>.21 ± .35</td>
<td>-.07 ± .04</td>
</tr>
</tbody>
</table>

$-2.2 \times 10^{-5}$ cm s$^{-1}$ at 1000 m, which is different from that for area B. The $w$-profile does not change much for $u_0$, $v_0$-values from other depth ranges in Table 10. The results for point E were $u_0 = 0.43 \pm 0.41$, $v_0 = -0.09 \pm 0.06$ cm s$^{-1}$ for depth range 200 to 1000 m. The results differed very much for the different depth ranges considered. This is
because many of the lines had small slopes against the $u_0$-axis, and the intersections were spread over a large area in the $u_0, v_0$-plane. Hence, we do not carry this analysis any further here.

5.3 The Meteor data

The east–west slopes $h_x$ were estimated visually on the sections of $\delta_{ST}$ along 8, 15, 22, 27, and 33°S appearing in Wüstr’s (1957) paper, at longitude 15°W (Fig. 19). The north–south slopes, $h_y$, at different points at 15°W were calculated from the meridional and vertical derivatives at that longitude. The slopes $h_x$ and $h_y$ do not show a smooth latitudinal dependence, which warns of some difficulties to be expected in the results. The reason is not likely to be measurement errors because the Meteor measurements were made with great care.

We tried different combinations of data. First, from the sections along 27 and 33°S (which do not run exactly along latitude circles) we calculated $h_x$ and $\delta_z$ as averages from these two sections and $h_y$ from the $\delta_y$ at 15°W and from $\delta_z$. The slopes and relative currents, down to 1500 m for this point, F, at 30°S, 15°W, are listed in Table 9. The relative current shows much weaker spiralling than that at point D but the sense of rotation is the same.

For point G (18.5°S, 15°W) the same calculation was done from sections along 15 and 22°S. Finally, for point H, we worked over a larger meridional distance, from 8 to 22°S, for the calculation of $h_y$. The relative currents for both points are similar and show only weak rotation to the right upwards (Fig. 25).

5.4 Results for points F, G, H

The lines in the $u_0, v_0$-plane for point F are shown in Fig. 26, where the reference depth is 1400 m. The lines are less regularly grouped than in previous graphs, because of the irregular
profiles of $h_x, h_y$ (Table 9). There is some tendency in the line intersections, though: lines 250 to 450 m intersect at $u_0 = 0.78 \pm 0.15$, $v_0 = -0.45 \pm 0.03$ cm s$^{-1}$, lines 650 to 850 m are almost parallel, and lines 950 to 1350 m intersect close to $v_0 = 0$, at $u_0 = 0.53 \pm 0.10$; $v_0 = -0.10 \pm 0.02$ cm s$^{-1}$. The result for the total depth range, 200 to 1400 m, is $u_0 = 0.60 \pm 0.43$, $v_0 = -0.22 \pm 0.06$ cm s$^{-1}$ where the error bars are corrected to an estimated NDF = 5 by
Both sides of the equations versus depth for point F.

Residuals, normalized by C, versus latitude for point F, depth ranges as indicated.
inspection of both sides of the equation in Fig. 27. In Table 10 the absolute currents are given for 1000 m instead of 1400 m for better comparison with the other data sets.

The residuals, normalized by C, versus latitude (Fig. 28) show a minimum at 10 S for the total depth range whereas for the depth range 600 to 1400 m the minimum is at the correct latitude. The good quality of the curve for that depth range seems to be caused by the fact that the significantly different lines 250 to 550 m were deleted in their calculation.

The vertical velocity calculated by use of the $u_0, v_0$ resulting from the fit to the total depth range is shown in Fig. 29, together with that from point D. In the deep part both profiles differ considerably but at 200 m we get $w = -1.5 \times 10^{-5}$ cm s$^{-1}$ for F compared with the $w = -2.0 \times 10^{-5}$ cm s$^{-1}$ for D.

It is of interest how the absolute currents at 1000-m depth at points D and F compare. At point D, we have $u_0 = 0.38 \pm 0.08$ cm s$^{-1}$ and at F, $u_0 = 0.23 \pm 0.43$ cm s$^{-1}$; for $v_0$ we have $v_0 = -0.18 \pm 0.04$ at D versus $v_0 = -0.11 \pm 0.06$ cm s$^{-1}$ at F. The values for these two mid-gyre points are the same within error bars.

At point G, the curve of residuals versus latitude for the total depth range 200 to 1500 m has two minima, but for an intermediate depth range of 400 to 1000 m the curve shows a single minimum close to the correct latitude. Again, as for point E, the lines have small slopes against the $u_0$ axis, which is because the $h_{yz}$ are small compared to $\beta/f$.

We mention only the result for the absolute currents at 1000 m, $u_0 = 0.21 \pm 0.35, v_0 = -0.07 \pm 0.04$ cm s$^{-1}$ for depth range 200 to 1500 m, which is not in disagreement with the results for point E at about the same latitude (Table 10). The resulting $w$-profile is taken up into Fig. 29. It does not differ much from the profile for point F except for the larger value $w = -3.1 \times 10^{-5}$ cm s$^{-1}$ at 200 m.

For point H, no reasonable $R/C(\phi)$ curves resulted.
6. The Subtropical North Pacific

6.1 Data selection, calculation of slopes and currents

The data sets analyzed in the previous parts all had the shortcoming of consisting only of single sections in an area, not even from the same season or year.

Now we are coming to the only data set we could get where we have some hope to eliminate eddy noise and maybe even the seasonal fluctuation. These are stations worked at monthly intervals in 1964 to 1965 by the R.V. Townsend Cromwell east of Hawaii, CHARNELL, AU and SECKEL (1967). We selected five stations consisting of a center station, C at 22°N, 151°W, Stas. N and S at 3° latitude north and south of it and stas. E and W 3° longitude east and west of it (Fig. 30). By averaging over 11 values at monthly intervals at each of these stations we calculate mean profiles of anomaly of specific volume and of anomaly of dynamic depth.

![Diagram](image)

Fig. 30. Positions of Townsend Cromwell stations used for analysis.

![Diagram](image)

Fig. 31. Geostrophic currents at 22°N, 151°W.
The slopes of isosteric planes were calculated from the horizontal $\delta$-gradients from south to north and from west to east and from the vertical $\delta$-gradient at C. It made almost no difference in the slope values when the vertical $\delta$-gradient was calculated from the average $\delta$-profile of Stas. N, E, S, W instead of Sta. C.

The geostrophic currents (Fig. 31) show a nice turn in the top level, but decrease almost as a straight line down to 1000 m below 300 m. The profile of $h_y - (\beta/f)z$ has a minimum at 400 m (Fig. 32) but $h_x$ also having a vanishing vertical derivative there means that no conclusions about levels of no motion are possible.

Fig. 33. The equations in the $u_0$, $v_0$ plane. Results with standard error bars for different depth ranges as indicated.
Fig. 34. Both sides of the equations versus depth.

Fig. 35. Residuals, normalized by $C$, versus latitude for depth ranges as indicated.
Slope errors were calculated from the variances of the horizontal and vertical gradients of $\delta$ and it turned out that only in the depth range of 300 to 500 m the slope values were larger than their error bars. We do not make use of the errors here and therefore we do not discuss them in more detail.

6.2 Results

The lines in the $u_0, v_0$ plane (Fig. 33) indicate that the deep levels, below 550 m, are not of much use for the analysis because the lines intersect at very oblique angles. The result for the total depth range, 100 to 1200 m, with an estimated NDF = 2 yields $u_0 = 2.75 \pm 0.48$, $v_0 = 0.39 \pm 0.12$ cm s$^{-1}$. The depth range 100 to 500 m yields a significantly larger $v_0$, namely $u_0 = 2.42 \pm 0.33$, $v_0 = 1.29 \pm 0.40$ cm s$^{-1}$. The deep levels lead to $u_0 = 1.01 \pm 1.30$, $v_0 = 0.18 \pm 0.14$ cm s$^{-1}$.

The residuals, normalized by $C$, show a minimum some 5 to 7° south of the correct latitude. When only the deep levels are taken, the minimum curve gets bad, which is to be expected considering the oblique lines and the large error in $u_0$.

We tried to find out why the minimum in Fig. 35 is displaced. We thought of the nonlinearity of $\beta/f$ over the meridional extent of the points and we then used meridional gradients only from N - C or only from C - S. It turned out, however, that the nonlinearity of $\beta/f$ was not the reason. Altogether this data set with repeated measurements at fixed positions did not yield a better quality analysis than the data sets previously discussed.

7. THE SUBPOLAR GYRE AT 55 N, 20 W

One can make a fair estimate of direction and magnitude of the slope of various $\sigma_t$ surfaces from the charts published by DIETRICH (1969). To estimate the slopes $h_x$ and $h_y$ at point C, which lies within the subpolar gyre of the North Atlantic (Fig. 1), the Dietrich plates were inspected in the region bounded by latitudes 50 and 60°N and longitudes 10 and 30°W. The resulting values of $h_x$, $h_y$, $h_z$, $u'$, $v'$ are given in Table 12 and Fig. 36. The relative spiral is shown in Fig. 2. It has a sense of rotation opposite to that observed in the subtropical northern hemisphere gyres.

The table shows us that the slopes are much larger in this spiral than they are in the subtropics: 1000 m in 1000 km, as compared to 10 to 50-m in 1000 km in the subtropics. The relative velocities are also greater. It also appears, by comparing the curves for $h_x$ and $h_y$, $-(\beta/f)z$ in Fig. 36, that the beta effect is quite small in this case, and it seems that the beta constraints cannot govern this spiral. Both $h_{xy}$ and $[h_y-(\beta/f)z]_z$ tend to vanish near 700 to 800 m, so that a good graphical determination of the zero of either $u$ or $v$ cannot be made. If the solution of the $u_0, v_0$ equations is carried out, we find vanishing velocities at mid-depth, and the method of wrong latitude gives us no minimum at the right latitude. Moreover, the computed values of $w$ are two orders of magnitude greater than expected. It seems that the true physical nature of this spiral is one in which water flows across isopycnals (low to high density) on the mean as a result of deep winter-time cooling each year. We know from observations at weather ships India and Juliett in this region that winter-time mixed layer depths extend to 600 and 400 m, respectively. As a consequence, it is certain that in the upper 600 m at least, the condition of no flow across density surfaces cannot be rigorously imposed.

It is therefore of some interest to explore the implications of a model in which the known mean surface flux of buoyancy can, when distributed over the upper 600 m, rotate the velocity vector. In very approximate terms, the computed heat loss from the surface of the ocean in
this region is approximately 100 g cal cm$^{-2}$ day$^{-1}$. There is presumably also a buoyancy gain due to excess of precipitation over evaporation in this region, but because the weather ships do not measure precipitation, we cannot evaluate this latter flux from actual measurements. However, to maintain the mean $T/S$ relation of the region, it may be expected to be roughly half the heat flux, so that we can compute the buoyancy flux by assuming that both heat loss and fresh water taken together are equivalent to about a total of 70 g cal cm$^{-2}$ day$^{-1}$. Ignoring $w$ in equation (1.4) and integrating over the top 600 m, we obtain:

$$\int_{-600}^{0} \frac{\gamma}{V^2} dz = \gamma\frac{H\rho_0}{c}$$

where $\gamma$ is the thermometric coefficient of expansion ($2 \times 10^{-4}$) and $H$ is the heat flux.

Table 11. Isosteric slopes and geostrophic currents in the subtropical North Pacific, at 22 N, 151 W.

<table>
<thead>
<tr>
<th>depth (m)</th>
<th>$h_x$ (m/1000 km)</th>
<th>$h_y$ (m/1000 km)</th>
<th>$u'$ (cm/sec)</th>
<th>$v'$ (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>73.7</td>
<td>70.2</td>
<td>-1.65</td>
<td>-2.43</td>
</tr>
<tr>
<td>200</td>
<td>31.6</td>
<td>7.6</td>
<td>-2.39</td>
<td>-1.65</td>
</tr>
<tr>
<td>300</td>
<td>33.6</td>
<td>-51.5</td>
<td>-2.08</td>
<td>-1.20</td>
</tr>
<tr>
<td>400</td>
<td>54.7</td>
<td>-96.3</td>
<td>-1.44</td>
<td>-0.80</td>
</tr>
<tr>
<td>500</td>
<td>51.3</td>
<td>-111.8</td>
<td>-0.83</td>
<td>-0.49</td>
</tr>
<tr>
<td>600</td>
<td>46.6</td>
<td>-96.1</td>
<td>-0.43</td>
<td>-0.28</td>
</tr>
<tr>
<td>700</td>
<td>46.5</td>
<td>-85.0</td>
<td>-0.13</td>
<td>-0.15</td>
</tr>
<tr>
<td>800</td>
<td>38.7</td>
<td>-37.1</td>
<td>0</td>
<td>-0.07</td>
</tr>
<tr>
<td>900</td>
<td>30.8</td>
<td>-23</td>
<td>0.01</td>
<td>-0.03</td>
</tr>
<tr>
<td>1000</td>
<td>16.0</td>
<td>61.7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1100</td>
<td>13.8</td>
<td>67.3</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>1200</td>
<td>10.1</td>
<td>75.5</td>
<td>-0.03</td>
<td>0.02</td>
</tr>
</tbody>
</table>
upwards out of the surface corresponding to a loss of 70 g cal cm\(^{-2}\) day\(^{-1}\). We also assume \(V \approx 1\) cm s\(^{-1}\) and \(\gamma = 10^7\) cgs. Thus,

\[
\theta_0 - \theta_{-600} = 1.4 \text{ radians.}
\]

This order of magnitude calculation tells us that the distributed buoyancy loss from the upper 600 m of the subpolar gyre can rotate the velocity vector by 1.4 radians, which, according to the observed relative spiral, is compatible with the degree of rotation observed.

### Table 12. Slopes and relative currents around 55°N, 20°W (area C).

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>(10^4 h_x)</th>
<th>(10^3 h_y)</th>
<th>(u') (cm/sec, rel 800 db)</th>
<th>(v') (cm/sec, rel 800 db)</th>
<th>(\theta_x)</th>
<th>(\theta_y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>141</td>
<td>890</td>
<td>1.25</td>
<td>2.64</td>
<td>912</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>-109</td>
<td>894</td>
<td>.86</td>
<td>2.58</td>
<td>927</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>-371</td>
<td>617</td>
<td>.57</td>
<td>2.22</td>
<td>861</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>-642</td>
<td>327</td>
<td>.32</td>
<td>1.73</td>
<td>382</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>-732</td>
<td>112</td>
<td>.14</td>
<td>1.22</td>
<td>178</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>.07</td>
<td>.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>-720</td>
<td>37</td>
<td>.0</td>
<td>.0</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

### 8.1 Relation to earlier studies

Two main features of our study are: (1) recognition of the existence of large spirals in the geostrophic flow regime of the central ocean, and (2) use of these spirals to compute the absolute velocity. So far as we know, the universality of the spirals in the open ocean has not hitherto attracted much attention, but of course, the question of determining the absolute velocity field has been a classic unsolved problem of physical oceanography. Fomin's (1964) book contains a helpful review of many of the earlier ideas about resolving this problem. Budget methods using (in addition to geostrophy) conservation of mass and tracer properties over certain volumes of the ocean have been used many times (Hidaka, 1940; Sverdrup, Johnson and Fleming, 1942; Riley, 1951), but these generally are severely limited by ill-conditionedness of the equation.

Defant's (1941) idea that the depth at which the vertical geostrophic shear vanishes is also the depth of vanishing velocity has never been supported with any clear dynamical reasons. Perhaps in a non-rotating deep lake, one could justify assuming that if the deep isopycnal surfaces have no slope over a large range of depth near the bottom, there is no horizontal motion there, but in a rotating system, this is not so obvious. In any case, if we apply the Defant criterion to the curves of \(h_x\) and \(h_y\) in the center of the North Atlantic gyre (our point B, Fig. 5a, line fix data), the depth where \(v\) vanishes is that at which \(h_y\) vanishes—which is below 1000 m. Slope \(h_y\) vanishes at approximately 900 m. The map (Fomin, 1964, Fig. 31) of the depth at which Defant claims both components \(u\) and \(v\) vanish simultaneously shows approximately 1100 m at our point B.

Parr (1938) proposed a different method, which in some respects is more similar to ours.
He suggests that if flow is along isopycnal surfaces, and geostrophic, a layer in which the density surfaces are nearly parallel to each other ('least distorted') is the most likely to be the level of vanishing velocity. This translates into our language as the level at which \( h_{zz} \) and \( h_{zz} \) vanish. We can read these depths from our Fig. 5 but we cannot attribute much significance to these numbers because Parr made no explicit use of the vorticity law, and hence did not include beta in his calculation. Furthermore, Parr's application of his method to strong Gulf Stream currents where relative vorticity may be important, and only the cross-stream equation of motion is geostrophic, means that we could not apply our simple method there. Despite the incompleteness of Parr's ideas, we find them interesting as an early approach to the method we present in this paper. The ideas of isopycnal flow, and the appeal to vertical variations in the horizontal gradient of the internal density structure of the flow have some similarity to what we have done.

Sverdrup's relation between the curl of the wind stress and the total vertically integrated transport in the interior of the ocean has been compared to the transport computed geostrophically (Leetmaa, et al., 1977) and suggests that the deep water velocities are zero within the errors of the comparison. The method applies only to the meridional component of velocity and not to the zonal component. At 28°N, Leetmaa et al. found the meridional component vanishes at 800 m, at 32°N at 1000 m. These are both deeper levels than obtained in our discussion of area B.

Stommel (1956) attempted to use the linear vorticity relation, and the meridional component of the geostrophic velocity computed at 32°N from the Atlantis stations that make up the western portion of the 32°N section (elsewhere occupied by the Discovery) to compute the value \( v_0 \). At the time, the full set of Discovery stations had not yet been occupied, and therefore, it was not possible to smooth out the eddies. As a result, the \( v \)-profile Stommel used is not representative of the average \( v \)-profile along 32°N that has been calculated here by smoothing. The vorticity equation was integrated vertically, using the computed Ekman pumping at the surface for the top boundary condition, and using the statement that \( w \) vanishes at the bottom as the bottom boundary condition. Possible complications due to bottom slope and a bottom frictional boundary layer were ignored. The depth of no motion that Stommel computed has no statistical significance, and in contrast to the method discussed in this paper depends upon uncertain estimates of the wind stress and effects of the bottom boundary layer.

To avoid the uncertainty of how to treat the bottom boundary layer, Sudo (1965) carried the integration by Stommel from the Ekman layer down to a sloping material surface, through which, by hypothesis, there was no flow. This was written in the form of a kinematic boundary condition, and hence determined a relation between the two absolute horizontal components of the velocity field. By repeating this calculation from the Ekman layer to a variety of different sloping surfaces, it was possible to find a number of independent relations for \( u_0, v_0 \) so that a solution could be obtained. Sudo did not attempt to smoothe out the eddies.

In our study, we decided to exclude the surface Ekman layer as a boundary condition altogether by integrating between two sloping surfaces within the thermocline, but we then discovered that it was more natural to make the calculation in differentiated form. We also decided that it was imperative to attempt to remove the eddy noise as much as possible by smoothing the data over many adjacent stations for determination both of the slopes of the density surfaces and for determination of the relative geostrophic velocity components.
9. SUMMARY AND CONCLUSIONS

Absolute profiles of zonal and meridional currents for different ocean regions are presented in Figs. 37 to 39. The currents $u_0, v_0$ at reference level are the best fit results for the total depth range analyzed in each case.

We have three data sets from gyre centers, point B in the North Atlantic (Fig. 3) and points D and F in the South Atlantic (Fig. 19). The levels of no zonal motion range from 550 to 750 m (Fig. 37), the levels of no meridional motion from 600 to 780 m. The zonal currents indicated at 1000 m are eastward and range from $0.22 \pm 0.08$ to $0.38 \pm 0.08$ cm s$^{-1}$. The meridional currents at 1000 m are poleward and range from $0.04 \pm 0.03$ to $0.18 \pm 0.04$ cm s$^{-1}$. The deep currents for points D and F in the South Atlantic calculated from data that were taken 30 years apart are the same within error bars.

We have already prescribed caution concerning accepting the detailed validity of these results, due to the described tendency of the lines in the $u_0, v_0$-plane to intersect closer to zero for deeper levels. On the other hand, the three different data sets yield mutually consistent results, and, for point D, the described tendency was only weak.

The downwelling velocities at 200 m in the different gyre centers (Figs. 10a, 29) calculated with the current profiles in Fig. 37 all fall within the narrow range from $-1.5$ to $-2.6 \times 10^{-5}$ cm s$^{-1}$.

We can compare two data sets in the North Equatorial currents: point A at 20°N, 54°W in the Atlantic (Fig. 3), and for a point at 22°N, 151°W in the Pacific (Fig. 30). Both zonal profiles show (Fig. 38) minimum velocities at 200 m, no westward flow below 100 m, and an eastward current at 1000 m of $4.38 \pm 0.26$ cm s$^{-1}$ (Atlantic) and $2.75 \pm 0.48$ cm s$^{-1}$ (Pacific). If only the depth range 600 to 1000 m is used, the eastward velocities at 1000 m will only be $0.92 \pm 0.37$ cm s$^{-1}$ (Atlantic) and $1.01 \pm 1.30$ cm s$^{-1}$ (Pacific). Then we would have levels of no zonal motion at 660 m (Atlantic) and 470 m (Pacific), and a westward flow with a velocity maximum at 200 m. The deep eastward flows do seem somewhat surprising and raise doubts about the application of our method to such largely zonal currents. The meridional currents are poleward at 1000 m, $0.79 \pm 0.19$ cm s$^{-1}$ in the Atlantic and $0.39 \pm 0.12$ cm s$^{-1}$ in the Pacific. Defant’s map of zero motion shows a depth of about 900 m at point A, whereas ours is much shallower.

The absolute profiles from the data below the South Equatorial current at points E, G in the South Atlantic (Fig. 19) are shown in Fig. 39. The test of residuals versus latitude was not good for these data. The zonal currents vanish at 670 m for both data sets and the currents at 1000 m are eastward. The meridional current vanishes at 660 m for point E, at 770 m for point G, and the flow is poleward at 1000 m. The two sets of data (Meteor and I.G.Y.) are from different years, but the curves in Fig. 39 are quite similar. Defant’s map yields a depth of 1100 m for zero motion, again in disagreement with our findings.

We have no explanation for the fact that in most of the regions investigated there is a range of depth at which both coefficients of equation (1.6) vanish simultaneously.

Several symptoms have been found that suggest that our simple physical model may not perfectly describe the physics of the thermocline in the central gyres: (i) the tendency for adjacent lines in the $u_0, v_0$ to intersect closer to the origin with increasing depth, (ii) the fact that the relative spirals change their sense of rotation less than the absolute spirals do, (iii) the result that the North Equatorial currents flow largely toward the east.

The cause of these symptoms might be (i) inadequacy of the data to describe the mean state, (ii) mixing across density surfaces, (iii) the neglect of meso-scale eddy transport in the vorticity equation. The first of these could be investigated most easily with standard
Fig. 37. Profiles of absolute currents in the center of the North Atlantic gyre (point B) and of the South Atlantic gyre (points D, F).

Fig. 38. Profiles of absolute currents below the North Equatorial currents in the Atlantic (point A) and in the Pacific.

Fig. 39. Absolute currents below the South Equatorial Current in the South Atlantic at points E, G.
techniques: a program of hydrographic or CTD stations with an eddy-resolving spacing in a
10° square near point B repeated as frequently as cruises of opportunity occur over a period of years.

Depending upon the results of such a program, we could then judge whether further
investigation into the detailed working of the Sverdrup mechanism in the central gyres of the ocean
should be later extended toward the surface to attempt a direct determination of the
divergence of the Ekman mass flux. We could also penetrate toward deeper water to
investigate the effects of bottom topography and of the benthic bottom layer.

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